Homework #5 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (20 points): Find the average potential over a spherical surface of radius R due to a point charge located inside the sphere, but not at the center.

Problem 2 (20 points): In 1-d, the functional form of the general solution to Laplace's equation is *V(x) = mx + b* (see Section 3.1.2).

2a (10 points): Find the functional form of the general solution to Laplace's equation in 3-d spherical coordinates for the case where V only depends on the radial coordinate *r*.

2b (10 points): Find the functional form of the general solution to Laplace's equation in 3-d cylindrical coordinates for the case where V only depends on the radial coordinate *s*.

Problem 3 (20 points): Consider an infinite grounded conducting plane with two charges above the plane: *-2q* at height *d*, and *+q* and height *3d*. Use image charges to determine the force on the upper charge (+q).

Problem 4 (40 points): Consider a point charge *q* at a distance *a* from the center of a grounded conducting sphere of radius *R* (with *a* > *R*), as in Example 3.2 in Griffiths.

4a (10 points): Use the law of cosines to show that you can write

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{r^2 + a^2 - 2racos\theta}} - \frac{q}{\sqrt{R^2 + (\frac{ra}{R})^2 - 2racos\theta}}\right]$$

4b (10 points): Use the boundary conditions on the electric field (and thus the normal derivative of *V*) at the surface of the sphere to find the induced surface charge density σ on the sphere, as a function of θ.

4c (10 points): Integrate the charge density over the surface of the sphere to find the total induced charge.

4d (10 points): Calculate the energy of this configuration by determining the work required to bring the charge *q* from infinity.